A2 only

Algebra II 6.1/2 Exploring Exponential Models.
Objectives: Determine the multiplier for exponential growth and decay; Write and evaluate exponential expressions to model growth and decay functions

Review Key Concepts: Exponential Functions $y = ab^x$

- If a>0 and b> 1, the function represents growth
- If a>0 and 0<b<1, the function represents decay

Exponential Growth and decay models:
Come in two forms. $y = a(1+r)^t$ And $y = a(1-r)^t$
$b = 1+r$ ab^t ab^t $b = 1-r$

y Final a initial r rate as decimal t ime

(1+r) or (1-r) are "b" in the model and represent the ratio between two consecutive y terms. They are the growth factor or decay factor
multiplier

Example 1. Find the multiplier (b) for each rate of exponential growth.
(1+r) or (1-r)

a. 9% growth	b. .18 % growth	c. Doubles	d. 9% decay	e. halfdecay
1.09	1.0018	100% 1+1 2	1-.09 .91	$\frac{1}{2} \to 50\%$ .5

Example 2 (6.1 ex4). Interpret an exponential function. A car was purchased for $24,000. The function $y = 24(0.8)^x$ can be used to model the value of the car (in thousands of dollars) *x* years after it was purchased.
Is this growth or decay? Decay $b = .8$

What is the rate of decay and what does it mean? .2
the car value decreases 20% each year
Graph on your calculator. What is the y intercept represent?
x=0 initial value of the car
What does the asymptote represent?
the value will approach 0 eventually
When will the value of the car be about $5000? (calc)
$y = 24(.8)^x$ after 7 years.

Example 3. (6.1 ex3) Model an exponential function.

The population of a large city was about 4.6 million in the year 2010 and grew at a rate of 1.3% for the next four years. 0.013

Write a model for the growth. let t = years since 2010 P in millions

$P = 4.6(1.013)^t$

What will the population be in 2040?

$P = 4.6(1.013)^{30} \approx 6.78$ million

You try.

A factory purchased a 3D printer in 2010. The value of the printer is modeled by the function $y = 30(0.93)^x$ where x is the number of years since 2010.

Is this growth or decay? $.07$ or 7%

What is the value of the printer after 10 years? $y = 30(.93)^{10} = \$14.52$

Does the printer lose more of its value in the first ten years or in the second ten years?

$30(.93)^{20} = 7.03$ lost more 1st ten years

Example 4. Compare average rates of change.

A museum purchased a painting and a sculpture in the same year. Their changing values are modeled as shown. Find the average rate of change of the value of each art work over the 5-year period. Which art work's value is increasing more quickly?

Sculpture Value

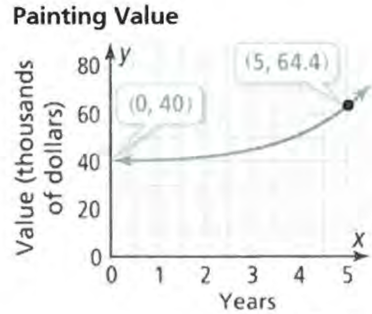
$f(x) = 50(1.075)^x$

(value in thousands of dollars in x years)

x	$f(x)$
0	50
5	91.78

$\frac{71.78 - 50}{5} = \$4.356/yr$

Painting Value



$\frac{64.4 - 40}{5}$

$4.88/yr$

greater

Day 2

The compound interest formula. $A = P(1+\frac{r}{n})^{nt}$

The total amount of an investment, A, earning compound interest is
Where A is Final amount, P is initial amount, r is rate as decimal,
n is number of compounds per year and t is years.

compounds: annual – 1, semiannual – 2, monthly → 12, quarterly → 4, weekly 52, daily 365

Example 5. Find the final amount of a $5000 investment after 3 years at 4% interest compounded annually, semiannually, quarterly, and monthly.

annually $A = 5000(1+\frac{.04}{1})^{1 \cdot 3} \approx \$5624.32$

semi $A = 5000(1+\frac{.04}{2})^{2 \cdot 3} \approx \$5630.81$

Quart. $A = 5000(1+\frac{.04}{4})^{4 \cdot 3} \approx \$5634.13$

Mon. $A = 5000(1+\frac{.04}{12})^{12 \cdot 3} \approx \$5636.36$

What is e?

Finding the Value of the "Natural Number"
Input the function $y = \left(1+\frac{1}{x}\right)^x$ into your calculator. What happens to your y-value as x gets larger? Use calc value:

x	$y = (1+\frac{1}{x})^x$
1	2
10	2.5937
100	2.7048
1000	2.7169

This number is called is called e ~ euler's number. or natural number

it is like π

NOTE: The natural number has the same properties as any other exponential function.

$e \approx 2.718281\ldots$
irrational

Evaluating e^x

How can you use a graphic calculator to calculate e^8? $e^{\frac{1}{2}}$? 1.649

2980.958

Continuously Compounded Interest. $A = Pe^{rt}$

A = final amount P= initial amount e=2.718... r= rate as decimal t= time in years or match rate

Example 6. (6.2 ex 4) Suppose you deposit $12,600 in an account paying 3.2% annual interest compounded continuously. Use the above formula to determine the amount *A*, you have in the account after 12 years.

$A = 12600e^{.032(12)} \approx \$18,498.63$

How long will it take for you to have at least $15000? $15000 = 12600e^{.032t}$

by calc

5.449 years

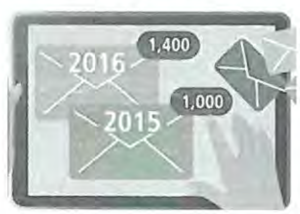
How long would it take for your money to double?

$25200 = 12600e^{.032t}$

21.661 years

Example 7. (6.2 ex 5) Using two points to find a model. $f(x) = ab^x$, "b" will be our growth or decay rate which is the ratio of our two y values.

Tia knew that the number of e-mails she sent was growing exponentially. She generated a record of the number of e-mails she sent each year since 2009. What is an exponential model that describes the data?



t = years from 2009

$r = \frac{1400}{1000} = 1.4$

$1400 = a(1.4)^7$

$a = 133$ → initial

$y = 133(1.4)^t$

You try. A surveyor determined the value of an area of land over a period of several years since 1950. The land was worth $31,000 in 1954 and $35,000 in 1955. Use the data to determine an exponential model that describes the value of the land.

t = years since 1950

$r = \frac{35000}{31000}$ $r = \frac{35}{31}$ ≈ 1.129

$31000 = a(35/31)^4$

$a \approx 19080.33$

$y = 19080.33(1.129)^t$